8 Diffraction

8.1 Diffraction phenomena

Consider that a beam of monochromatic light from a distant source passes through a narrow horizontal slit and is then intercepted by a viewing screen. If light traveled in straight lines as rays, then the slit would merely allow some of those rays through and they would form a sharp, bright rendition of the slit on the viewing screen. However, in fact, the light produces on the screen an interference pattern like that in Fig. 1. This pattern consists of a broad and intense central maximum and a number of narrower and less intensive maxima to both sides. In between the maxima are minima. Such pattern is called **diffraction pattern.**

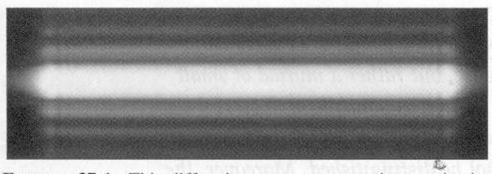

Fig. 1 Diffraction pattern appeared on a viewing screen when light that had passed through a narrow horizontal slit reached the screen.

Fig. 2 shows a diffraction pattern of a circular aperture, that is, a circular opening. Note the central maximum and the circular secondary maxima.

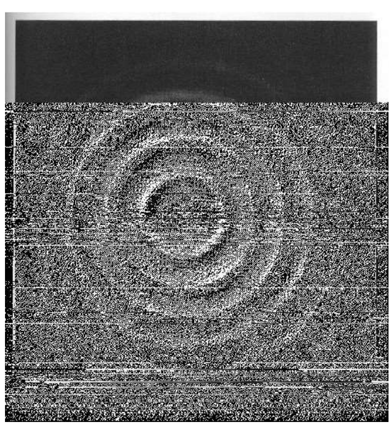

Fig. 2 Diffraction pattern of a circular aperture.

There is no significant physical distinction between *interference* and *diffraction*. It has, however, become somewhat customary, if not always appropriate, to speak of interference when considering the superposition of only a few waves and diffraction when treating a large number of waves.

So, diffraction of light is actually the deviation of light from rectilinear propagation. The diffraction pattern is characteristic of the diffraction of light. We must conclude that geometrical optics is only an approximation.

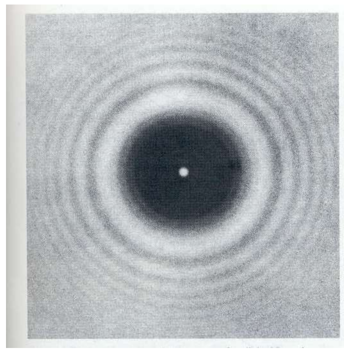

Fig. 3 Diffraction pattern of a disk.

Diffraction can be divided into two kinds in terms of the ratio of a/R, where a is the dimension of an aperture and R is the distance between the aperture and the observation point. When $a^2/R \ll \lambda$, it's Fraunhofer diffraction; otherwise, it's Fresnel diffraction.

Consider the configuration in figure 4 for Fraunhofer diffraction . A monochromatic plane wave propagating in the x-direction is incident on the opaque diffraction plate Σ. The electric field at a far-field point P(X,Y,Z) is, according to the Huygens-Fresnel principle, contributed by the wavelets reradiated from the points all over the aperture.

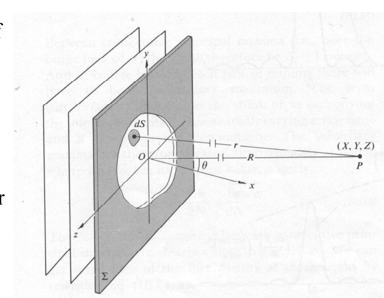

Figure 4 Fraunhofer diffraction by an aperture

$$E(P) = \iint_{aperture} \frac{\varepsilon(y,z)}{r} e^{i(\omega t - kr)} ds \qquad (8.1)$$

Where $\varepsilon(y,z)$ is the surface strength per unit area, k is the wave (propagation) number. The distance from dS located at (0,y,z) to point P located at (X, Y, Z) is r.

$$r = [X^2 + (Y-y)^2 + (Z-z)^2]^{1/2} \qquad (8.2)$$

For Fraunhofer diffraction, the r in the denominator of (8.1) can be replaced by the distance from the center O to the point P. That is

$$r \approx R = [X^2 + Y^2 + Z^2]^{1/2} \qquad (8.3)$$

The r in the exponent should be more accurate since k is usually large. Expanding (8.2) and using (8.3), we have

$$r = R[1+(y^2+z^2)/R^2 - 2(Yy+Zz)/R^2]^{1/2}$$
$$= R[1-2(Yy+Zz)/R^2]^{1/2}$$
$$= R[1-(Yy+Zz)/R^2]$$
$$= R-(Yy+Zz)/R \qquad (8.4)$$

Furthermore, $\varepsilon(x,y)=\varepsilon_A$ for small aperture. The total electric field is then

$$E = \frac{\varepsilon_A e^{i(\omega t - kR)}}{R} \iint_{aperture} e^{ik(Yy+Zz)/R} dS \qquad (8.5)$$

The irradiance (Intensity) is $I = EE^*/2$

8.2 Single slit Fraunhofer diffraction

The slit is very long along the y direction so the diffraction pattern should be identical along Y direction at least near the screen center. Along the z direction, the width is b. Let's consider, in the paper plane, a point P that has a distance Z from the center line. Eq. (8.5) becomes

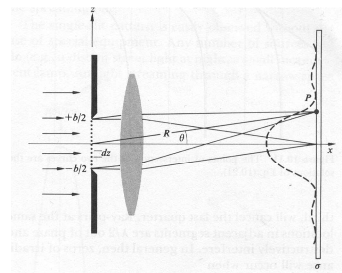

Figure 5 Single slit diffraction

$$E = \frac{\varepsilon_A e^{i(\omega t-kR)}}{R}\int_{-b/2}^{b/2} e^{izkZ/R} dz$$
$$= \frac{\varepsilon_A e^{i(\omega t-kR)}}{R(ikZ/R)} e^{izkZ/R}\Big|_{-b/2}^{b/2} = b\frac{\varepsilon_A e^{i(\omega t-kR)}}{R}\frac{e^{ikbZ/2R}-e^{-ikbZ/2R}}{2i(kbZ/2R)}$$
$$= b\frac{\varepsilon_A e^{i(\omega t-kR)}}{R}\frac{\sin\beta}{\beta} \qquad (8.6)$$

$$with\ \ \beta = kbZ/2R = (kb/2)\sin(\theta) \qquad (8.7)$$

$$I(\theta) = EE^*/2 = \frac{1}{2}(\frac{\varepsilon_A b}{R})^2(\frac{\sin\beta}{\beta})^2 \qquad (8.8)$$

When $\theta = 0, \sin\beta/\beta = 1,\ \mathrm{I}(0) = \frac{1}{2}(\frac{\varepsilon_A b}{R})^2$, So

$$I(\theta) = I(0)(\frac{\sin\beta}{\beta})^2 \qquad (8.9)$$

Eq. (8.9) is shown in figure 4 and 1. Zeros occurs when $\beta = m\pi, (m=\pm1,\pm2..)$The first zero occurs at $\beta=\pi$,or $\sin\theta_1 = \lambda/b$. The smaller of the slit width, the larger of the central maximum range.

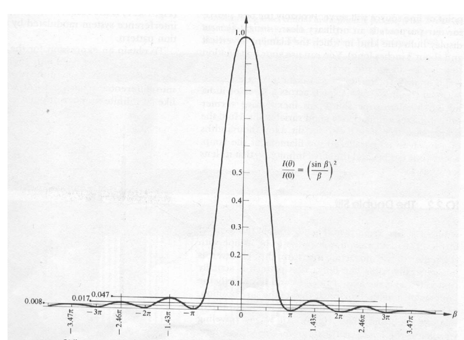

Figure 6 Fraunhofer diffraction pattern of a single slit